Unit IV: The Refraction of Light

Fast. It's Fast.

For ages, light was thought to travel instantaneously from one place to the next. There were a few attempts to measure the speed of light however:

Take one:
Galileo, in the 17th century, was the first person to try to measure the speed of light. He and his assistant stood on hilltops one mile apart with lanterns. Galileo would flash his light at his assistant, who opened a shutter on his lantern as soon as he saw Galileo's light.

Galileo timed how long it took to see his assistant's light. However, he found that the time of the event was too small to measure.

Take two:
Ole Romer during the 1670's made careful observations of Jupiter's moon Io. Since Io has a very stable orbit, Romer concluded he should be able to predict where Io is in its orbit. However, he noticed that sometimes the moon was behind schedule and sometimes it was ahead, depending on the time of the year.

Romer discovered that Io was ahead of schedule when Earth was closest to Jupiter, and behind when they were farthest apart. This is due to the fact that light has to travel farther when Earth and Jupiter are far apart.

His calculations showed that light travelled at 300 000 000 m/s, or 3.00×10^8 m/s.

Although with better equipment we have measured the speed of light to be 299 792 456.2 m/s we still use Romer's 3.0×10^8 m/s as a good approximation. We will use the following equation when dealing with the speed of light:

$$c = \frac{d}{t} \qquad v = \frac{d}{t}$$

where:
c = the speed of light (3×10^8 m/s)
d = distance travelled (m)
t = time (s)

Therefore, if we know the distance we can solve for the time it takes light to travel that distance. Likewise, if we know the distance light travels we can find out how much time it takes to do so.

One thing to mention is that since time must be in seconds, we should brush up on conversions.

1 min = 60 s	1 hour = 60 min	1 day = 24 h 1 year = 365.25 days

Ex. How many seconds are in 2.35 years?

$$\frac{2.35\text{ years}}{1}\left(\frac{365.25\text{ days}}{1\text{ year}}\right)\left(\frac{24\text{ h}}{1\text{ day}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 74160360\text{ s}$$

≈ 74200000 s

$\boxed{\approx 7.42 \times 10^7 \text{ s}}$

Ex. How long does it take light to travel from earth to a space ship that is 7.80 x 10^{10} km away?

$t = ?$
$v = 3.00 \times 10^8 m/s$
$d = 7.80 \times 10^{10} Km = 7.80 \times 10^{13} m$

$v = \frac{d}{t}$
$t = \frac{d}{v}$
$t = \frac{7.80 \times 10^{13} m}{3.00 \times 10^8 m/s}$

$t = 260\,000 s$
$\boxed{t = 2.60 \times 10^5 s}$

Ex. A space ship orbiting a distant planet sends out a distress call via radio. How far is the receiving ship away if they get the message 1.20 hours after it was sent?

$d = ?$
$t = 1.20 \text{ hours} = 4320 s$
$v = 3.00 \times 10^8 m/s$

$v = \frac{d}{t}$
$d = vt$

$d = (3.00 \times 10^8 m/s)(4320 s)$
$d = 1\,296\,000\,000\,000 m$
$\boxed{d = 1.30 \times 10^{12} m}$

Things are Far Away

Since astronomers deal with such vast distances, sometimes they use a distance called a **light year**. A light year is the **distance** light travels in 1 year. Note that it is **not** a measure of time!

What is the distance of a light year?

$d = ?$
$v = 3.00 \times 10^8 m/s$
$t = 1 year \left(\frac{365.25 days}{1 year}\right)\left(\frac{24 hours}{1 days}\right)\left(\frac{3600 s}{1 hour}\right) = 31557600 s$

$v = \frac{d}{t}$
$d = vt$

$d = (3.00 \times 10^8 m/s)(31557600 s)$
$d = 9.46728 \times 10^{15} m$
$\boxed{d = 9.46 \times 10^{15} m}$

Another handy distance is an **astronomical unit (A.U.)**. An astronomical unit is the distance between the sun and the earth.

What is the distance, in km, for 1 A.U. if it takes light 8.33 minutes to get from the sun to earth?

$t = 8.33 min = 499.8 s$
$v = 3.00 \times 10^8 m/s$
$d = ?$

$v = \frac{d}{t}$
$d = vt$

$d = (3.00 \times 10^8 m/s)(499.8 s)$
$d = 1.4994 \times 10^{11} m$
$\boxed{d = 1.50 \times 10^{11} m}$

Another unit for distance is the **parsec**. We will not concern ourselves with its definition as it is beyond the span of this course. Just know:

1 pc = 3.26 ly

We will convert distances through dimensional analysis, just as we do with time conversions.
How many kilometers are in 1 parsec?

$\frac{1 pc}{1}\left(\frac{3.26 ly}{1 pc}\right)\left(\frac{9.46 \times 10^{15} m}{1 ly}\right)\left(\frac{1 Km}{1000 m}\right) \Rightarrow 1 pc = 3.08396 \times 10^{13} Km$

$\boxed{1 pc \simeq 3.08 \times 10^{13} Km}$

Ex. A star is 8.52 ly away. How many years does it take for light to reach us from this star? How many parsecs is this star away from earth?

~~$d = 8.52\,ly = 8.05992 \times 10^{16} m$~~

$\boxed{t = 8.52 \text{ years}}$

ly → pc

$\frac{8.52\,ly}{1}\left(\frac{1\,pc}{3.26\,ly}\right)$

$= 2.6134...$

$\boxed{8.52\,ly \approx 2.61\,pc}$

Ex. How many seconds does light travel for if it travels a distance of 6.45 light years?

$t = 6.45$ years

$\frac{6.45 \text{ years}}{1}\left(\frac{365.25 \text{ days}}{1 \text{ year}}\right)\left(\frac{24\,h}{1\,d}\right)\left(\frac{3600\,s}{1\,h}\right)$

$t = 203546520\,s$

$\boxed{t \approx 2.04 \times 10^{8} s}$

It Bends?

The substance light is travelling in is called a **medium**.

When light travels from one media to another it bends. This bending of light is known as **refraction.** The bending of light is due to the fact that light travels at different speeds in different media. Therefore, each substance has its own unique speed of light.

For example, light travels faster in air than in water.

The fastest light can travel is in a vacuum. This is basically 3.00 x 10^8 m/s. The next fastest is air, which we approximate at 3.00 x 10^8 m/s, although it **does** travel slower than in a vacuum.

Each medium has its own **index of refraction (n)**, which is a comparison of the speed of light in that particular substance, with the speed of light in a vacuum.

Note that there is a list of indices of refraction on the back of our formula sheet.

$$n = \frac{c}{v}$$

where:
n = index of refraction
c = speed of light in a vacuum (m/s)
v = speed of light in a medium (m/s)

Ex. What is the index of refraction of glass if the speed of light in glass is 2.00 x 108m/s?

$v = 2.00 \times 10^8 m/s$
$c = 3.00 \times 10^8 m/s$
$n = ?$

$n = \frac{c}{v}$

$n = \frac{3.00 \times 10^8 m/s}{2.00 \times 10^8 m/s}$

$\boxed{n = 1.50}$

Ex) Find the speed of light in water.

$v = ?$
$c = 3.00 \times 10^8 m/s$
$n = 1.33$

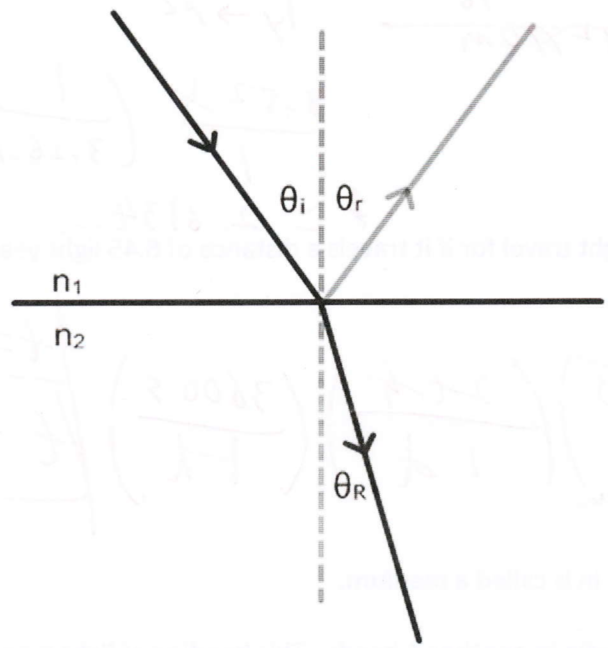
$n = \frac{c}{v}$

$v = \frac{c}{n}$

$v = \frac{3.00 \times 10^8 m/s}{1.33}$

$\boxed{v \approx 2.26 \times 10^8 m/s}$

Ray diagram depicting refraction:



The point at which two media meet is called an **interface**.

When light hits an interface, it bends at the interface, but not in one material or the other. This happens at the **point of incidence**.

The line perpendicular to the interface running through the point of incidence is still called a **normal**.

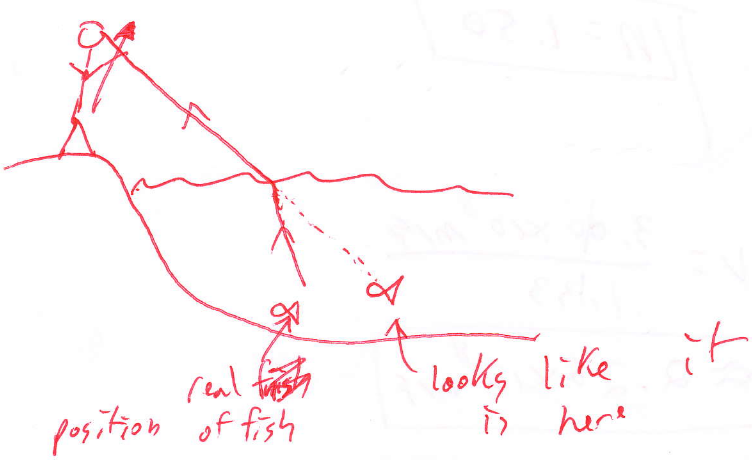
The angle between the incident ray and the normal is the **angle of incidence**.

The refracted ray is the ray that leaves the interface through the second medium.

When an incident ray strikes an interface, most of the light will be refracted. The angle between the refracted ray and the normal is the **angle of refraction**.

Some of the light will get reflected. The angle between this reflected ray and the normal is still called the angle of reflection.

Spear Fishing:



Snell that? That's the Snell of Science.

There are two laws of refraction.

The first law of refraction is known as **Snell's Law** and is in the form of the following equation:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

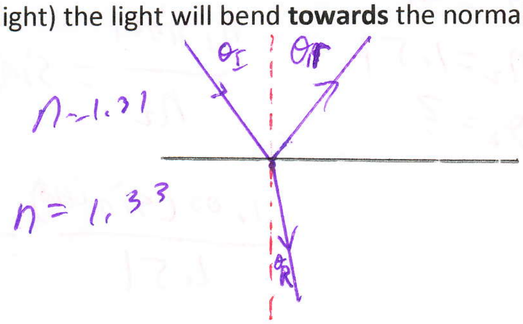
Where:

n1 = incident substance
θ1 = angle of incidence
n2 = refractive substance
θ2 = angle of refraction

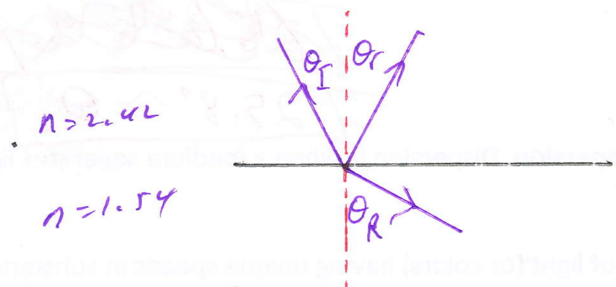
The second law of refraction states that the incident and refracted rays are on opposite sides of the interface, and opposite sides of the normal.

We will use Snell's law extensively to study how light behaves at an interface.

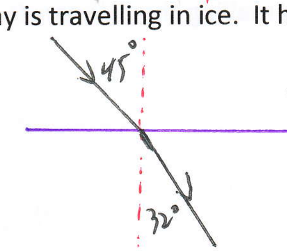
If light is travelling from less refractive medium (small n, fast speed of light) to a more refractive medium (big n, slow speed of light) the light will bend **towards** the normal.



If light is travelling from a more refractive medium to a less refractive medium then light will bend **away** from the normal.



Ex. An incident light ray is travelling in ice. It hits an ice-beryllium glass interface at 45.0°. What is the angle of refraction?



Ice 1.31

beryllium glass 1.75

$\theta_1 = 45.0°$
$n_1 = 1.31$
$n_2 = 1.75$
$\theta_2 = ?$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{n_1 \sin\theta_1}{n_2} = \sin\theta_2$

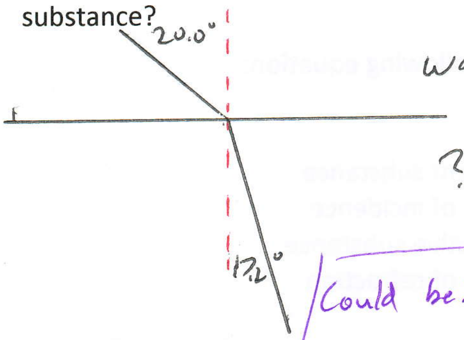
$\frac{1.31 \sin 45°}{1.75} = \sin\theta_2$

$\sin\theta_2 \approx 0.52932$

$\theta_2 = \sin^{-1}(0.52932)$

$\boxed{\theta_2 \approx 32.0°}$

Ex. A cube of an unknown substance is submerged in water. If an incident ray travelling through the water strikes the unknown substance at 20.0° and produces an angle of refraction of 17.2°, what is the unknown substance?



$n_1 = 1.33$

$\theta_1 = 20.0°$

$\theta_2 = 17.2°$

$n_2 = ?$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{n_1 \sin\theta_1}{\sin\theta_2} = n_2$

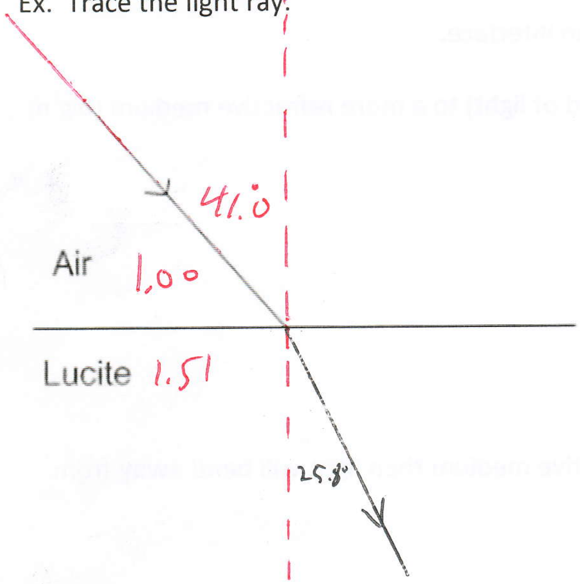
$\frac{1.33(\sin 20.0°)}{\sin(17.2°)} = n_2$

$1.538246947 = n_2$

$1.54 \approx n_2$

Could be: Ruby
Crystal Glass
Crystal Quartz

Ex. Trace the light ray.



$n_1 = 1.00$

$\theta_1 = 41°$

$n_2 = 1.51$

$\theta_2 = ?$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{n_1 \sin\theta_1}{n_2} = \sin\theta_2$

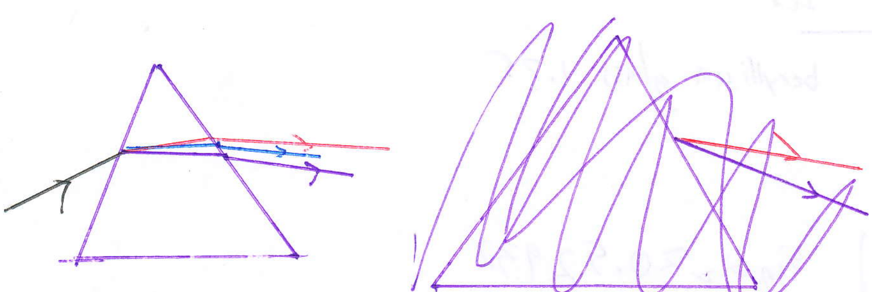
$\frac{1.00(\sin 41.0)}{1.51} = \sin\theta_2$

$0.43448 \approx \sin\theta_2$

$\boxed{25.8° \approx \theta_2}$

One phenomenon related to refraction is called **dispersion**. Dispersion is when a medium separates light into different colors on the spectrum.

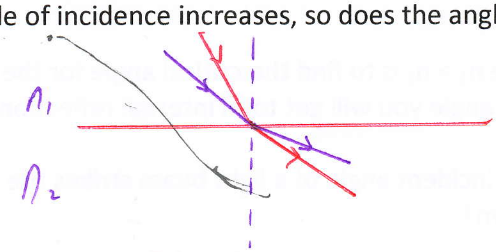
This phenomenon is due to different wavelengths of light (or colors) having unique speeds in substances. Therefore, each color of light will bend at a slightly different angle, making them separate.



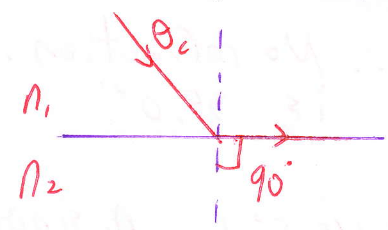
Total. Internal. Reflection.

The angle of refraction depends on two things: the substances involved, and the angle of incidence.

If the incident substance is more refractive than the second substance, then the ray will refract away from the normal. Therefore, as the angle of incidence increases, so does the angle of refraction.



If this is the case, then there should be an incident ray which will produce an angle of refraction of 90°.

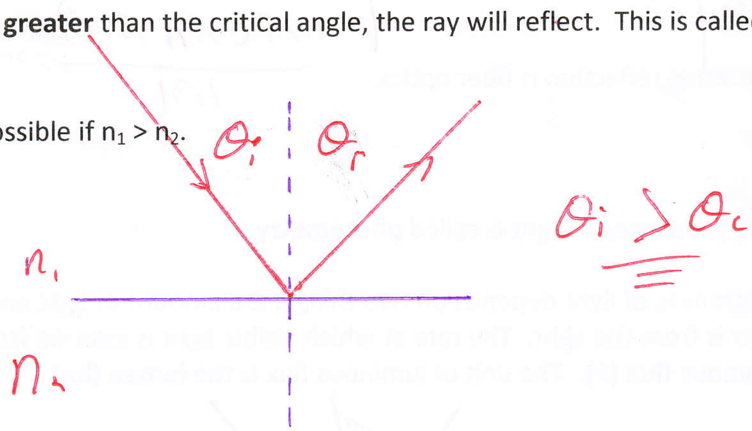


The **critical angle** for two substances is an angle of incidence that produces an angle of refraction of 90°. This is always the same for two substances.

If the angle of incidence is **greater** than the critical angle, the ray will reflect. This is called total **internal reflection**.

Remember - this is only possible if $n_1 > n_2$.

$n_1 > n_2$



Snell's law can be used to find the critical angle.

Ex. Calculate the critical angle between glycerin and water.

$\theta_c = ?$
$n_1 = 1.47$
$n_2 = 1.33$
$\theta_2 = 90°$

$n_1 \sin\theta_c = n_2 \sin\theta_2$

$\sin\theta_c = \dfrac{n_2 \sin\theta_2}{n_1}$

$\sin\theta_c = \dfrac{1.33 \sin(90°)}{1.47}$

$\sin\theta_c = \dfrac{1.33}{1.47}$

$\boxed{\theta_c \approx 64.8°}$

Note that since the critical angle always produces an angle of refraction of 90°, we can clean up Snell's law for this situation:

$$\sin\theta_c = \frac{n_2}{n_1}$$

Note: $n_1 > n_2$

At the critical angle, the refracted ray will travel parallel to the interface. The greater the difference is between the indices of refraction, the smaller the critical angle.

For example, the critical angle between diamond and air is smaller than the critical angle between diamond and water.

A good idea when doing problems where $n_1 > n_2$ is to find the critical angle for the two substances. Then, if your incident angle is greater than the critical angle you will get total internal reflection.

Ex. What is the angle of refraction if the incident angle of a light beam strikes the following interfaces at 35.0°. If it reflects, what is the angle of reflection?

a) zircon - air

$\theta_R = ?$
$n_1 = 1.92$
$n_2 = 1.00$
$\theta_c = ?$
$\theta_1 = 35.0°$

$\sin\theta_c = \frac{n_2}{n_1}$

$\sin\theta_c = \frac{1.00}{1.92}$

$\theta_c = 31.4°$

$31.4° < 35°$

∴ No refraction. The angle of reflection is 35.0°.

b) ice - beryllium glass

n_2 n_1

$\theta_2 = ?$
$n_1 = 1.75$
$n_2 = 1.31$
$\theta_c = ?$
$\theta_1 = 35.0°$

$\sin\theta_c = \frac{n_2}{n_1}$

$\sin\theta_c = \frac{1.31}{1.75}$

$\theta_c = 48.5°$

$35.0° < 48.5°$

∴ Refraction

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{n_1 \sin\theta_1}{n_2} = \sin\theta_2$

$\frac{1.75(\sin 35.0°)}{1.31} = \sin\theta_2$

$\boxed{\theta_2 = 50.0°}$

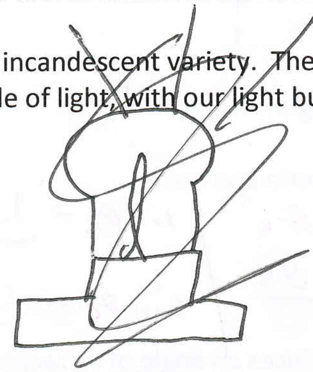
A practical use of total internal reflection is fiber optics.

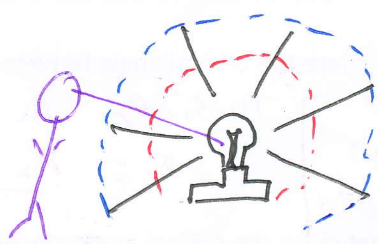
Light Me Up

The science of measuring properties of light is called **photometry**.

We all know that the brightness of light depends on two things: the amount of light produced from the light, and the distance the observer is from the light. The rate at which visible light is emitted from a source, or the power output, is called the **luminous flux (P).** The unit of luminous flux is the **lumen (lm)**.

Most lights we will deal with will be of the incandescent variety. These light bulbs emit light in all directions. Therefore, we could imagine a sphere made of light, with our light bulb at the center of the sphere.





A standard 100 W incandescent light bulb will emit 1750 lm, meaning that if the light were at the center of a sphere then 1750 lm indicates all of the light that strikes the inside surface of the sphere during a given time period.

We are usually more concerned with the amount of light that falls on a particular area, like a book or a desk. The amount of light that falls on a given area is called **illuminance (E)**. The unit of illuminance is lm/m^2. However, physicists call 1 lm/m^2 a **lux (lx)**.

$$E = \frac{P}{4\pi r^2} = \frac{P}{AREA}$$

Surface area of a sphere

Where:
E = illuminance (lm/m^2) (lx)
P = luminous flux (lm)
r = distance object is from light (m)

Ex. What is the illuminance of a desk sitting 2.50 m below a 1750 lm incandescent light bulb?

$E = ?$
$r = 2.50\ m$
$P = 1750\ lm$

$$E = \frac{P}{4\pi r^2}$$

$$E = \frac{1750\ lm}{4\pi (2.50\ m)^2}$$

$$\boxed{E = 22.3\ lx}$$

Ex. How far away is a light source if an object has an illumination of 90.0 lx under a light producing 1500 lm?

$E = 90.0\ lx$
$P = 1500\ lm$
$r = ?$

$$E = \frac{P}{4\pi r^2}$$

$$Er^2 = \frac{P}{4\pi}$$

$$\sqrt{r^2} = \sqrt{\frac{P}{4\pi E}}$$

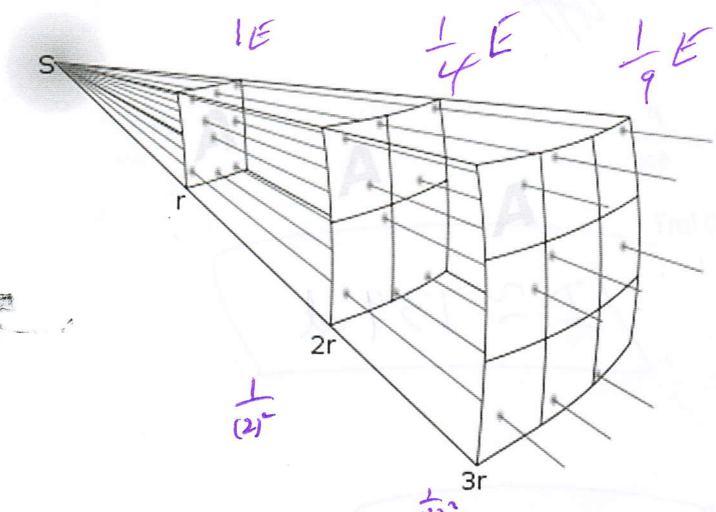
$$r = \pm\sqrt{\frac{P}{4\pi E}}$$

$$r = \pm\sqrt{\frac{1500\ lm}{4\pi\, 90.0\ lx}}$$

$r = \pm 1.151647165\ m$

$$\boxed{r \approx 1.2\ m}$$

As a surface gets farther and farther from a light source, it becomes less illuminated. In fact, the illuminance of a surface varies inversely with the square of the distance from the light source.



$$E \propto \frac{1}{d^2}$$

So, if you double the distance from a light source, then the illuminance is cut down to ¼. If you triple the distance, then the surface has only 1/9 the illuminance.

To compare two light sources, we can use this inverse-square relationship. First, find out how many times farther one object is from the source compared to the other. Then, use the inverse square relationship.

Ex. How much more illuminance does object A have sitting 3.00 m from a 1750 lm source than object B sitting 12.0 m from the source? After, compare object B to A.

A to B

$d = \frac{3.00m}{12.0m} = \frac{1}{4}$

$E = \frac{1}{d^2}$

$E = \frac{1}{(\frac{1}{4})^2}$

$E = \frac{1}{\frac{1}{16}}$

$\boxed{E = 16}$

B to A

$d = \frac{12.0m}{3.0m} = 4$

$E = \frac{1}{d^2} = \frac{1}{4^2} = \boxed{\frac{1}{16}}$

Ex. What is the comparison if we changed the light bulb from 1750 lm to 2500 lm?

The same

So, to summarize, if you wanted to increase the illumination of an object you could increase the output of the light source, or decrease the distance between the light and the object.

Intensity

The **intensity** of a light source is the luminous flux that falls on a square meter of a sphere with a radius of 1m. In other words, intensity is a measure of the **brightness** of the bulb. So, if you had an intense light bulb, it would be very bright 1 m away from it when you compare to a dim bulb from the same distance.

The unit for intensity is the **candela (cd)**. This is derived from the old unit for intensity - candle power.

Since our radius of the 'light sphere' emitted by the bulb is 1 m, then we get the following equation for intensity:

$I = \frac{P}{4\pi r^2}$ $\quad I = \frac{P}{4\pi (1m)^2}$ $\quad I = \frac{P}{4\pi}$

$r = 1m$

$$I = \frac{P}{4\pi}$$

Ex. What is the luminous intensity of a bulb with 1750 lm?

$I = ?$

$P = 1750\ lm$

$I = \frac{P}{4\pi}$

$I = \frac{1750\ lm}{4\pi}$

$\boxed{I \simeq 139\ cd}$

Ex. What is the luminous flux of a 1.5 cd light bulb?

$I = 1.5\ cd$

$P = ?$

$I = \frac{P}{4\pi}$

$4\pi I = P$

$4\pi (1.5\ cd) = P$

$\boxed{P \approx 19\ lm}$

Another equation that involves intensity is:

$$E = \frac{I}{d^2}$$

Where:
E= illuminance (lx)
I = intensity (cd)
d = distance from source (m)

Ex. What is the illumination of an object sitting 2.00 m from a bulb with an intensity of 2.75 cd?

E = ?
I = 2.75 cd
d = 2.00 m

$E = \frac{I}{d^2}$

$E = \frac{2.75\,cd}{(2.00\,m)^2}$

$\boxed{E \approx 0.688\ lx}$

There are two trickier situations we can introduce now. The first is when we have two objects at different distances from the **same** light bulb. The key for this situation is to notice that the light bulb will have the same **intensity** since the properties of the bulb do not change.

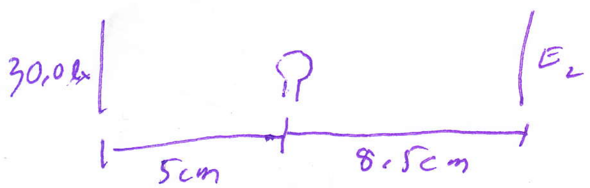
Mathematically, we get:

$E_1 = \frac{I}{d_1^2}$ $\quad E_2 = \frac{I}{d_2^2}$ $\quad I = I$

$\therefore E_1 d_1^2 = I$ $\quad \therefore E_2 d_2^2 = I$ $\quad E_1 d_1^2 = E_2 d_2^2$

$$E_1 d_1^2 = E_2 d_2^2$$

Ex. A screen 5.00 cm away from a bulb has an illumination of 30.0 lx. Another screen is 8.50 cm away from the same bulb. What is the illumination on the second screen?



$E_2 = ?$
$E_1 = 30.0\,lx$
$d_1 = 5\,cm = 0.05\,m$
$d_2 = 8.5\,cm = 0.085\,m$

$E_1 d_1^2 = E_2 d_2^2$

$\frac{E_1 d_1^2}{d_2^2} = E_2$

$\frac{(30.0\,lx)(0.05\,m)^2}{(0.085\,m)^2} = E_2$

$\boxed{E_2 \approx 10.4\,lx}$

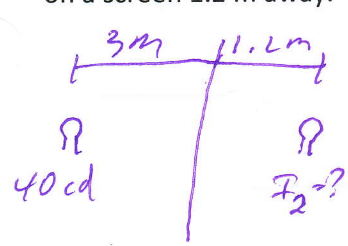
The other situation is when we have different light sources providing the same **illumination** on an object. Each light source will have its own intensity, so mathematically:

$E = \frac{I_1}{d_1^2}$ $\quad E = \frac{I_2}{d_2^2}$

$\therefore E = E$

$$\frac{I_1}{d_1^2} = \frac{I_2}{d_2^2}$$

Ex. Bulb A, with a luminous intensity of 40.0 cd has the same illuminance on a screen 3.0 m away as bulb B does on a screen 1.2 m away. What is the luminous intensity of bulb B?



$I_1 = 40\,cd$
$d_1 = 3\,m$
$I_2 = ?$
$d_2 = 1.2\,m$

$\frac{I_1}{d_1^2} = \frac{I_2}{d_2^2}$

$\frac{I_1 d_2^2}{d_1^2} = I_2$

$\frac{40\,cd\,(1.2\,m)^2}{(3\,m)^2} = I_2$

$\boxed{6.4\,cd \approx I_2}$